Solving equations

Solving an equation means finding the value or values for which the two expressions on each side of the equals sign are equal. One of the most common methods used to solve equations is the **balance method**.

Imagine an equation as a set of scales. The scales will stay in balance as long as the same operation (addition, subtraction, multiplication or division) is applied to both sides.

Example

Solve the equation 3a + 8 = 26.

The equation can be shown in balance on a set of scales.

The value of a must be found that balances 3a + 8 with 26.

The term +8 can be removed from the equation by subtracting 8 from each side. This gives 3a + 8 – 8 = 26 – 8.

This simplifies to 3a = 18. 3a means 3 x a, so to get a by itself, divide by 3. This gives a = 18 ÷ 3.

This simplifies to a = 6.

This answer can be checked by substituting a = 6 back into the original equation 3a + 8 = 26.

3a + 8 = 3 x 6 + 8 = 18 + 8 = 26

The equation balances, so a = 6 is the correct answer.

Eg. Solve the equation 4y + 5 = -3

We need to leave 4y on its own, so we subtract 5 from both sides.

4y + 5 **– 5** = -3 **- 5**

4y = -8

Dividing by 4,

y = -8 ÷ 4

y = -2

To check if the answer is correct replace y with -2 in the equation 4y + 5 = -3.

4 x -2 + 5 = -8 + 5 = -3. Therefore, the answer is correct.

Solving equations with brackets

Eg. Solve the equation 5(2c – 3) = 19

The equation contains a set of brackets. The easiest way to solve equations with brackets is to expand the brackets by multiplying the number outside the brackets by each term inside the brackets.

5(2c – 3) = 19

5 x 2c – 5 x 3 = 19

10c – 15 = 19

Leave 10c on its own by adding 15 to each side:

10c – 15 **+ 15** = 19 **+ 15**

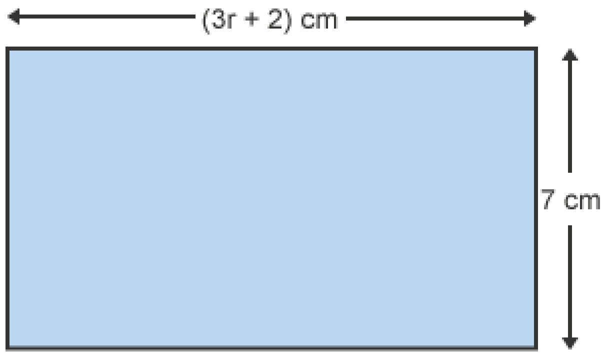
10c = 34

Dividing by 10,

c = 34 ÷ 10

c = $^{34}/_{10}$ = $^{17}/_{5}$ or 3.4

Eg. The area of this rectangle is 56 cm^2. Find the value of r.



To find the area, we multiply the length by the width which means that we need to multiply 3r + 2 by 7

This can be written as 7(3r + 2). Since the area is equal to 56 cm^2, we can write,

7(3r + 2) = 56.

7 x 3r + 7 x 2 = 56

21r + 14 = 56

Subtracting 14 from both sides to leave 21r on its own,

21r + 14 **- 14** = 56 **– 14**

21r = 42

r = 42 ÷ 21 = 2

Solving equations with unknowns on both sides

Some equations have letters on each side of the equals sign, for example: 5(k - 3) = 2(k + 6).

Solve this equation by rearranging all the letters onto one side of the equation and all numbers onto the other side. The easiest way to do this is usually by moving the unknown with the smallest coefficient in the equation (the letter with the smallest number in front of it).

Eg. Solve 5(k - 3) = 2(k + 6).

Expand the brackets:

5k - 15 = 2k + 12

Decide which of the unknowns has the smallest number in front of it. 2 is less than 5 so subtract 2k from each side.

5k - 15 **– 2k** = 2k + 12 **– 2k**

3k - 15 = 12

Adding 15 to both sides in order to leave 3k on its own,

3k - 15 **+ 15** = 12 **+ 15**

3k = 27

Dividing by 3,

$k = 27 \div 3$

$k = 9$

Eg. Solve $5d - 12 = 8d + 10$

Decide which of the unknowns has the smallest number in front of it. 5 is less than 8 so subtract 5d from each side.

$5d - 12$ **– 5d** $= 8d + 10$ **– 5d**

$-12 = 3d + 10$

To leave 3d on its own, subtract 10 from both sides

-12 **- 10** $= 3d + 10$ **– 10**

$-22 = 3d$

If $3d = -22$

$d = -22 \div 3 = -\frac{22}{3}$ or $-7\frac{1}{3}$

Eg. Solve $8 - 3m = 10 - 5m$

Decide which of the unknowns has the smallest number in front of it. -5 is less than -3 so add 5m to each side.

$8 - 3m$ **+ 5m** $= 10 - 5m$ **+ 5m**

$8 + 2m = 10$

To leave 2m on its own, subtract 8 from both sides

$8 + 2m \mathbf{- 8} = 10 \mathbf{- 8}$

$2m = 2$

$m = 2 \div 2 = 1$