

Ratio and proportion

Ratios are usually written in the form $a:b$ and can be used on maps to show the scale in relation to real life. Two quantities are in direct proportion when they increase or decrease in the same ratio.

<p>Purple paint is made by mixing blue and red paint in the ratio of 2 to 3.</p>  <p>2:3</p>	<p>To make mortar, sand and cement are mixed together in the ratio of 5 to 2.</p>  <p>5:2</p>	<p>Lilly, Jack and Jo have shared the money in the ratio of 2 to 6 to 3.</p>  <p>2:6:3</p>
<p>A ratio must be written in the correct order, with the quantity mentioned first written first.</p> <p>The ratio of cats to dogs is 3:4. ✓ <u>NOT</u> The ratio of dogs to cats is 3:4. ✗</p> <p>Note that the ratio of dogs to cats is 4:3.</p> 		

Equivalent ratios and simplest form

If you are making orange squash and you mix one part orange to four parts water, then the ratio of orange to water will be 1:4.

If you use 1 litre of orange, you will use 4 litres of water (1:4).

If you use 2 litres of orange, you will use 8 litres of water (2:8).

If you use 10 litres of orange, you will use 40 litres of water (10:40).

These ratios are all equivalent:

$$1:4 = 2:8 = 10:40$$

Both sides of the ratio can be multiplied or divided by the same number to give an equivalent ratio.

Example Write the ratio 40:28 in its simplest form.

To work this out, look for a number that will divide exactly into 40 and 28.

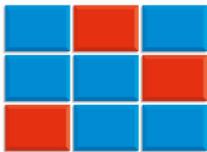
2 divides into both numbers, so 40:28 can be written as 20:14. But you can divide these once again by 2, so the simplified ratio is 10:7.

No number divides into 10 and 7 exactly, so 10:7 is the simplest form of the ratio.

Simplifying Ratios

Ratios are easier to work out when they are in their simplest form.

To simplify ratios, both numbers must be **divided by their highest common factor**.



The ratio of blue to red tiles is 6 to 3 but this can be simplified. 3 is the highest common factor of 6 and 3, so divide both numbers by 3.



Simplify these ratios:

a) 20 : 16

We can divide both by 4

So the answer is 5 : 4

b) 21: 30

We can divide both by 3

So the answer is 7 : 10

c) 10 : 25 : 40

Although we now have three items being compare together, we still use the same method.

5 is a common factor, so we divide by 5.

The answer would be 2 : 5 : 8

d) 12 m : 18 m

If the ratio includes units, we could still simplify it and while simplifying, we remove the units as long as they are the same.

Both are in metres and the largest common factor is 6.

Therefore if we divide by 6, the answer would be 2 : 3

e) €28 : €35

Both are in euro and the largest common factor is 7

Therefore if we divide by 7, we get 4 : 5

f) 320 cm : 4 m

In this case, to simplify we first need to change one of the items, so that both have the same units.

4 m is equal to 400 cm and therefore the ratio could be written as:

320 cm : 400 cm

Now that the units are the same, we could divide by a common factor.

We can start by dividing by 10 and get:

32 : 40

Then we can divide by 8 and the answer is 4 : 5

Dividing in a given ratio

Example Dave and Lisa win €500 between them. They agree to divide the money in the ratio 2:3. How much does each person receive?

The ratio 2:3 tells us that for every 2 euro that Dave receives, Lisa will receive 3 euro.

In other words, this tells us that, of the total amount, Dave will receive 2 parts and Lisa will receive 3 parts. This means that there are 5 parts in total.

€500 represents 5 parts. Therefore, to find the value of one part, we divide €500 by 5.

$$€500 \div 5 = €100$$

Dave receives 2 parts: $2 \times €100 = €200$

Lisa receives 3 parts: $3 \times €100 = €300$

REMEMBER It's important to notice what order the parts of the ratio are written in. 2:3 is not the same as 3:2.

In this example, the ratio of Dave's money to Lisa's was 2:3. If we swap the order to 3:2 then Dave would get more than Lisa.

To keep it the same as in the example we could say that the ratio of Lisa's money to Dave's would be 3:2.

Example A necklace is made using gold and silver beads in the ratio 3:2. If there are 80 beads in the necklace:

- a) How many are gold? b) How many are silver?

Gold : Silver = 3:2, so there are 5 parts altogether.

$80 \div 5 = 16$, so 1 part represents 16 beads.

a) Gold = $3 \times 16 = 48$ beads.

b) Silver = $2 \times 16 = 32$ beads.

Dividing in a Ratio

Sometimes an amount needs to be divided according to a particular ratio.

Ava, Isla and Freya made €315 selling balloons at a fayre. They agreed to split the money in the ratio of 3:2:4. How much money does each person get?



1 Add the numbers in the ratio to calculate the total number of parts.

$$3 + 2 + 4 = 9$$

2 Find the value of 1 part by dividing the total amount by the total number of parts, 9.

$$315 \div 9 = 35$$

1 part = 35

3 Multiply the value of 1 part, 35, by the numbers in the ratio to calculate how much money each person gets.

$$\begin{aligned} 3 \times 35 &= 105 \\ 2 \times 35 &= 70 \\ 4 \times 35 &= 140 \end{aligned}$$

4 315 divided in the ratio of 3:2:4 is 105:70:140. Check your answer by adding together the values.

Ava	Isla	Freya
€105	€70	€140
$105 + 70 + 140 = 315$		

Finding one quantity of a ratio, given the other

If you know the value of one part of a ratio, you can calculate the values of the other parts, and the total sum of the ratio.



To make turquoise paint, blue paint and green paint are mixed in the ratio of **4:7**. If Eva has **2.4 litres of blue paint**, how much green paint does she need and how much turquoise paint can she make?

1

Calculate the value of **1 part** by dividing the amount of blue paint by the number of blue parts in the ratio.

$$2.4 \div 4 = 0.6$$

1 part = 0.6

2

To calculate the amount of green paint that is needed, multiply the value of **1 part** by the number of green parts in the ratio.

$$0.6 \times 7 = 4.2$$

Eva needs 4.2 litres of green paint.

3

To calculate the amount of turquoise paint that Eva can make, multiply the value of **1 part** by the total number of parts in the ratio.

$$0.6 \times 11 = 6.6$$

Eva can make 6.6 litres of turquoise paint.

We can also work it out this way:

$$\begin{array}{r} \text{Blue : Turquoise} \\ 4 : 7 \\ 2.4 : ? \end{array}$$

We can work out this using the so called cross-multiplication:

$$2.4 \times 7 = 16.8 \div 4 = 4.2 \text{ litres}$$

Example Amy and Sharon win a sum of money, which they agree to share in the ratio 5:3. If Amy receives €150, how much will Sharon receive?

Amy receives 5 parts, which is equivalent to €150. Therefore, €150 represents 5 parts.

€150 ÷ 5 represents one part, so one part is €30.

Sharon receives 3 parts: 3 x €30 = €90.

We can also reason this out by saying

Amy : Sharon

5 : 3

€150 : ?

$$150 \times 3 = 450 \div 5 = \text{€}90$$