Probability

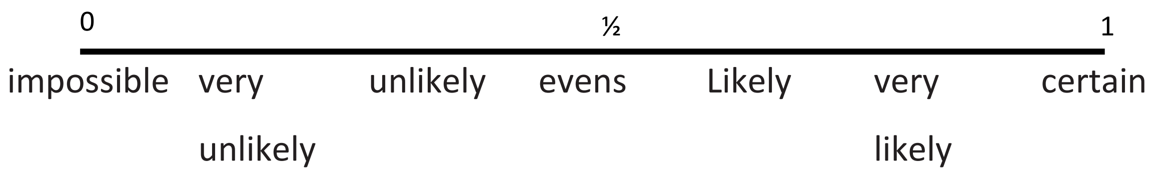
Probability is a measure of chance. It tells us how likely an event is to happen. We can use words such as certain, likely, evens and impossible to describe the likelihood of events.

If an event is certain to occur, that means it must happen 100% of the time. An example of this would be getting a number less than 7 on a six-sided dice. This is certain as every number on a standard dice is less than 7.

Usually, we denote probabilities in fraction form. This means that if something is 'certain' to happen, it would have a probability of 1. No event can ever have a probability greater than 1.

Similarly, an 'impossible' event can never occur, such as the chance of pulling a black ball from a bag of green balls. It simply cannot happen. This is a probability of 0.

The terms 'likely' and 'unlikely' mean a probability of greater than and less than ½ respectively. We can show this information on the probability diagram below which is called the probability scale:



Roll of a dice

We talked about the probability of rolling a dice and getting a number less than 7 and decided it was certain since all numbers on a dice are less than 7.

What about the probability of getting a number 4 on a single roll of a dice? If we know that the dice is fair, the probability of getting a 4 is $^{1}/_{6}$. This is because we are looking for one specific number out of a total of 6 numbers.

The probability of getting any single number on a normal dice is $^{1}/_{6}$.

If we want to find the probability of getting an even number, we know that between 1 and 6 there are three even numbers (2, 4 and 6) and therefore the probability is $^{3}/_{6} = {}^{1}/_{2}$.

What if we had a 20-sided dice, and we wanted to know the probability of getting a number less than 5? There are 4 numbers less than 5 (1,2,3,4) and the dice has 20 numbers in total. The probability is therefore $^{4}/_{20} = {}^{1}/_{5}$.

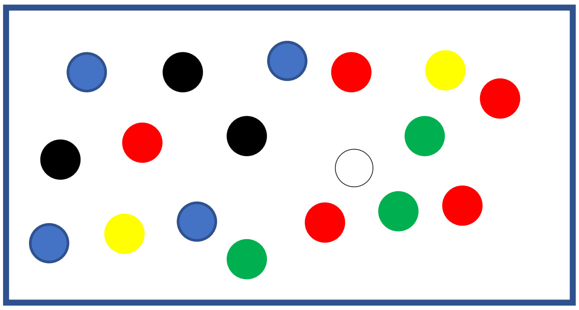
If we are looking for the probability of **not** getting a number less than 5 (getting a number greater than or equal to 5), there are 16 numbers and so the probability is $^{16}/_{20} = {}^{4}/_{5}$.

We can notice that if we add together the probabilities of getting a number less than 5 and not getting a number less than 5, the answer is 1.

$^{1}/_{5} + {}^{4}/_{5} = 1$

Example Mike is going to pick one ball from this box. What is the probability of:

a) Picking a blue ball
b) Picking a white ball
c) Picking a yellow ball
d) Picking a red ball
e) Picking a black ball
f) Picking a green ball
g) Picking a pink ball
h) **Not** picking a red ball
i) Picking red or blue



In all there are 18 balls: 5 red, 4 blue, 3 green, 3 black, 2 yellow and 1 white.

a) Since there are 4 blue balls out of a total of 18, the probability of picking a blue ball is $^4/_{18}$, which can be simplified to $^2/_9$.
b) Since there is 1 white ball out of a total of 18, the probability of picking a blue ball is $^1/_{18}$.
c) Since there are 2 yellow balls out of a total of 18, the probability of picking a blue ball is $^2/_{18}$, which can be simplified to $^1/_9$.
d) The probability of picking a red ball is $^5/_{18}$.
e) The probability of picking a black ball is $^3/_{18}$, which can be simplified to $^1/_6$.
f) The probability of picking a green ball is $^3/_{18} = ^1/_6$.
g) The probability of picking a pink ball is 0 because there are no pink balls.
h) To find the probability of NOT picking a red ball, you must add all the other balls (13) and the probability would be equal to $^{13}/_{18}$.
i) To find the probability of picking red or blue, you need to add the number of red and blue balls (5 + 4 = 9) and the probability would be equal to is $^9/_{18} = ^1/_2$.

Possibility Space

Possibility space is a term used in mathematics to mean all possible outcomes. For example, the possibility space for rolling a normal dice is {1,2,3,4,5,6} as these are all the only outcomes we can obtain.

The possibility space for flipping a coin is {H, T}.

What if you have two coins?

When two coins are flipped, the possible outcomes are {H,H}, {H,T}, {T,H} and {T,T}.

This could be better shown in a table:

		Second coin	
		H	T
First coin	H	H,H	H,T
	T	T,H	T,T

We can see that there are four possible outcomes.

The probability of getting two Heads is therefore ¼.

The probability of getting a Heads and a Tails is $^{2}/_{4}$ or ½ .

What if we wanted to know the possible outcomes for flipping a coin and rolling a dice?

The possibility space for these two combined events is {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}.

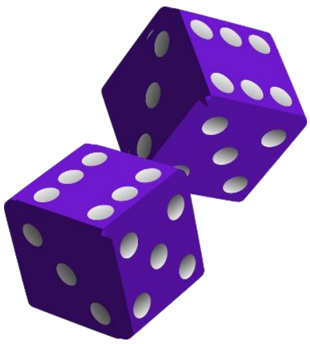
This can also be written in the form of a table:

	1	2	3	4	5	6
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

There are 12 possible outcomes. So if we wanted to find the probability of getting a Heads and a 3, this is $^{1}/_{12}$, since a Heads and a 3 together occur only once.

On the other hand, the probability of getting a Heads and an even number is $^{3}/_{12}$ (or $^{1}/_{4}$), because there are {H,2} {H,4} and {H,6}

We could also write out the possibility space for rolling two dice, but to simplify things we use possibility space tables.



				2nd dice			
		1	2	3	4	5	6
	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
1st dice	3	3,1	3,2	3,3	3,4	3,5	3,6
	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

A possibility space diagram showing the possible outcomes of rolling two dice. It has 6 rows and 6 columns.

From the diagram, we can see that there are 36 possible outcomes.

a) What is the probability of getting two 6s?

   There is only one {6,6} and therefore the probability of getting 6,6 is $^1/_{36}$

b) What is the probability of getting a similar score on the two dice?

   There is {1,1} {2,2} {3,3} {4,4} {5,5} and {6,6} and therefore the probability of getting a similar score on the two dice is $^6/_{36}$, which can be simplified to $^1/_6$.

c) What is the probability that prime numbers appear on both dice?

   Prime numbers are 2, 3 and 5. We therefore have {2,2} {2,3} {2,5} {3,2} {3,3} {3,5} {5,2} {5,3} and {5,5}. Therefore, the probability is $^9/_{36}$, which can be simplified to $^1/_4$.

Example Elise rolls two dice and records the product of the two scores (the two scores multiplied by each other).

2nd dice

1st dice

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

There are 36 possible outcomes.

a) What is the probability of getting an even number?

There are 27 even numbers in the table. Therefore, the probability is $^{27}/_{36}$, which can be simplified to $^{3}/_{4}$.

b) What is the probability of getting a square number?

There are 8 square numbers in the table and therefore the probability is $^{8}/_{36}$, which can be simplified to $^{2}/_{9}$.

Example A four-sided spinner has the numbers 1 to 4 marked on it. It is spun twice and the two scores are noted. Draw the possibility space to show the outcomes. Find the probability that:

a) The total score is even

b) The two separate scores are even

c) The product of the scores is even

		2nd spinner			
		1	2	3	4
1st spinner	1	1,1	1,2	1,3	1,4
	2	2,1	2,2	2,3	2,4
	3	3,1	3,2	3,3	3,4
	4	4,1	4,2	4,3	4,4

The total possible outcomes is 16.

a) Below are the ones which would give an even total score.

		2nd spinner			
		1	2	3	4
1st spinner	1	1,1	1,2	1,3	1,4
	2	2,1	2,2	2,3	2,4
	3	3,1	3,2	3,3	3,4
	4	4,1	4,2	4,3	4,4

The probability is therefore $^{8}/_{16}$, which can be simplified to ½

b) Below are the ones where the scores are both even.

		2nd spinner			
		1	2	3	4
1st spinner	1	1,1	1,2	1,3	1,4
	2	2,1	2,2	2,3	2,4
	3	3,1	3,2	3,3	3,4
	4	4,1	4,2	4,3	4,4

The probability is therefore $^{4}/_{16}$, which can be simplified to ¼

c) Below are the ones where the product of the scores is even.

		2nd spinner			
		1	2	3	4
1st spinner	1	1,1	(1,2)	1,3	(1,4)
	2	(2,1)	(2,2)	(2,3)	(2,4)
	3	3,1	(3,2)	3,3	(3,4)
	4	(4,1)	(4,2)	(4,3)	(4,4)

The probability is therefore $^{12}/_{16}$, which can be simplified to $^{3}/_{4}$