Transformations

Shapes can be transformed in a number of ways. These include translation, rotation and reflection.

Translation: When translating a shape, you can move it up or down or from side to side, but you cannot change its appearance in any other way. When a shape is translated, each of the vertices (corners) must be moved in exactly the same way.

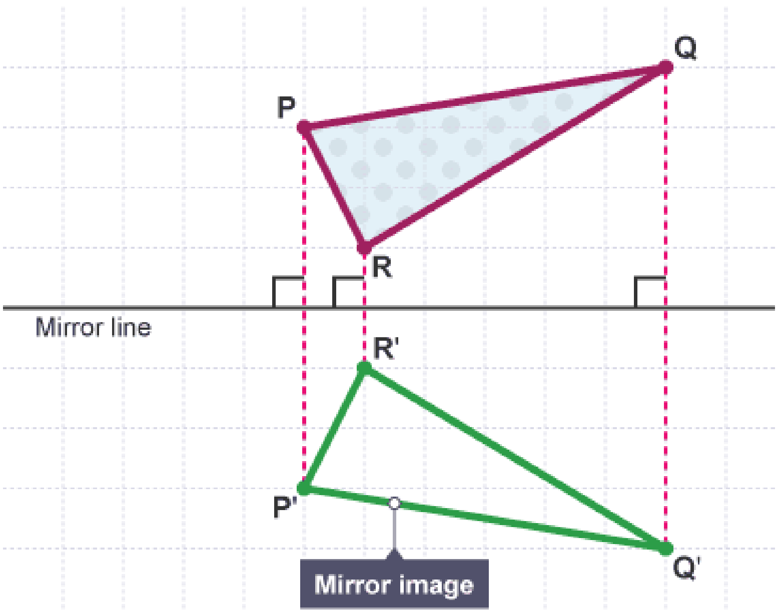
Rotation: If you put a sheet of paper on a table and place your pen in the middle of it, you can rotate the paper whilst keeping the pen in a fixed position. The pen is acting as a centre of rotation and you can rotate an object around it, anywhere between 0° and 360°.

Reflection: If you look in a mirror, you see your own image. You (the object) and your image appear to be the same distance and angle from the mirror. The mirror acts as a line of reflection, through which your image is copied.

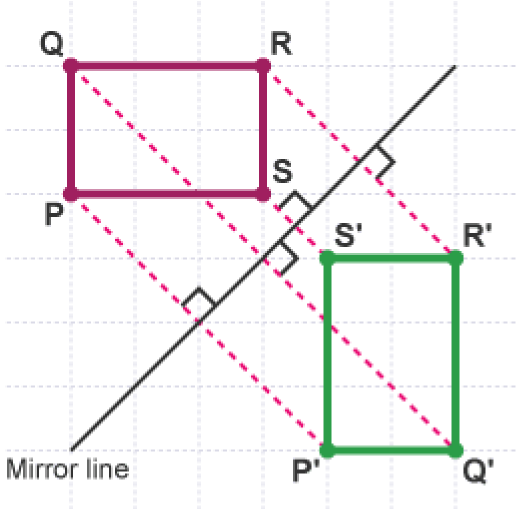
Reflection

A shape can be reflected across a line of reflection to create an image. The line of reflection is also called the mirror line.

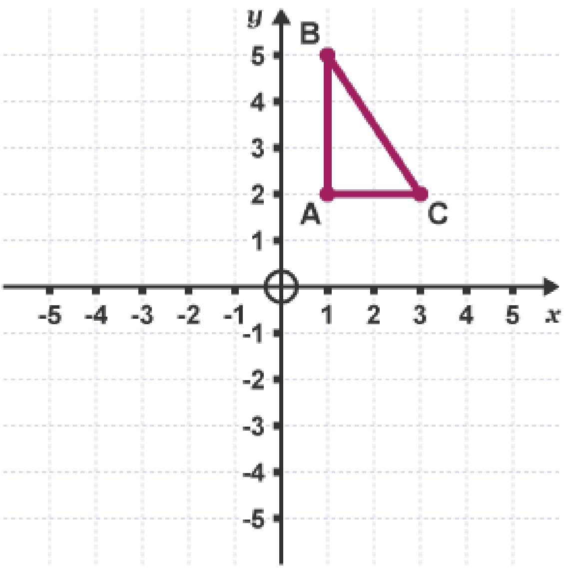
The triangle PQR has been reflected in the mirror line to create the image P'Q'R'.



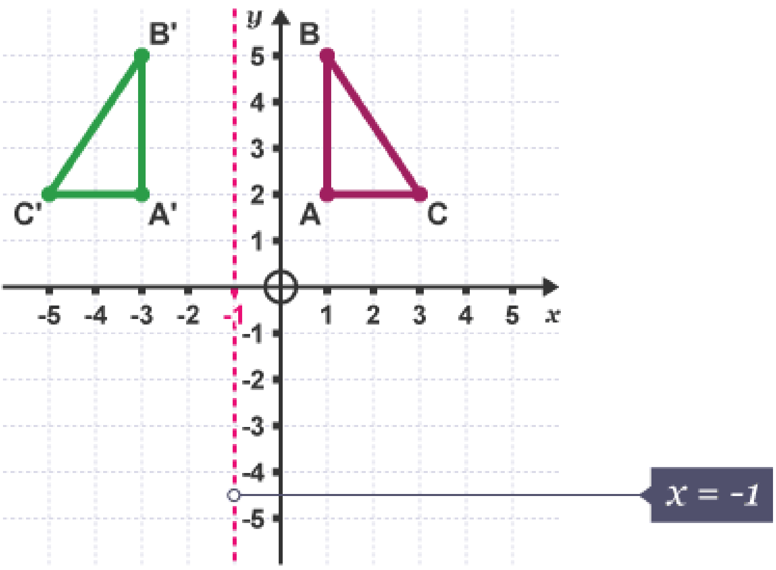
Every point in the image is the same distance from the mirror line as the original shape.



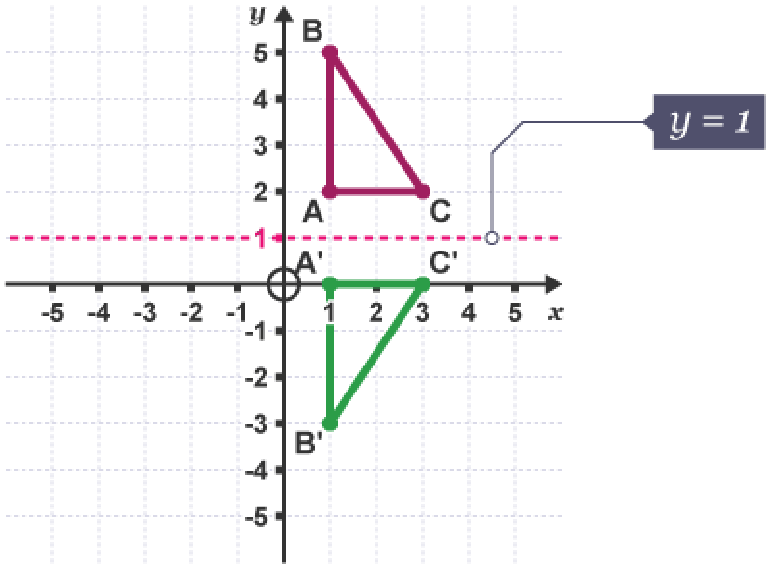
Example Reflect the shape in the line x = -1.



The line x = -1 is a vertical line which passes through -1 on the x-axis.



Example Describe the transformation of the shape ABC.

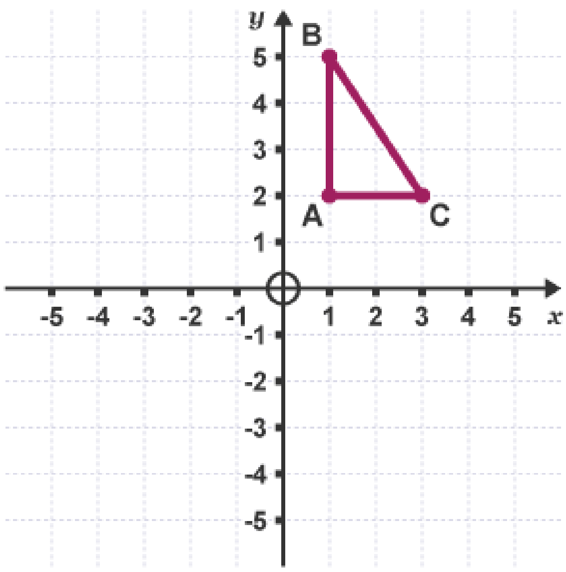


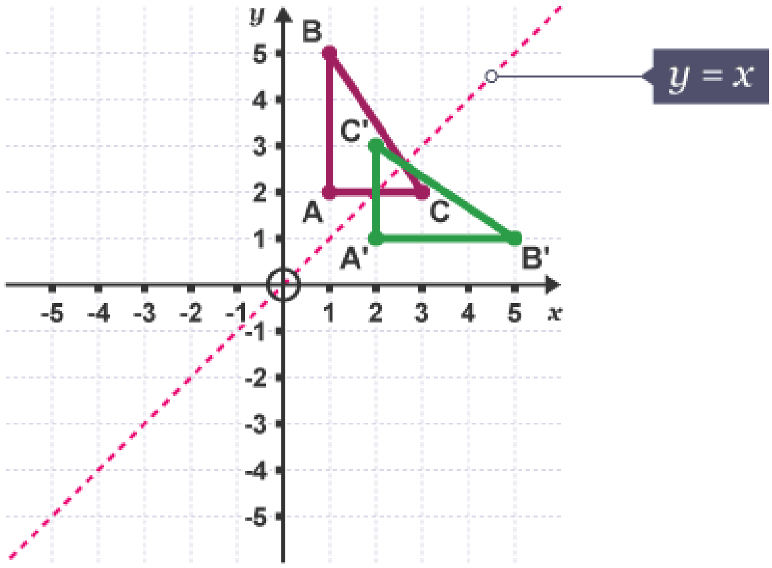
The line y = 1 is a horizontal line which passes through 1 on the y-axis.

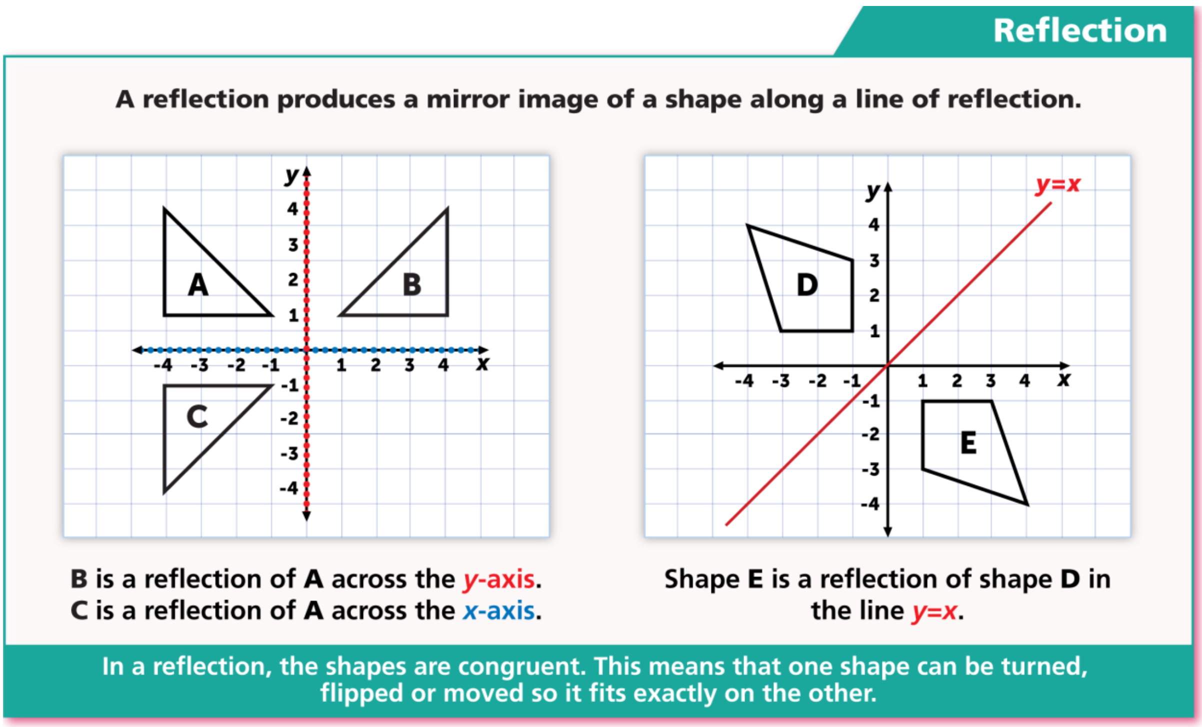
The shape is a reflection in the line y = 1.

Example Reflect the shape in the line y = x.

The line y = x passes through points that have the same x and y coordinates, eg (0, 0), (1, 1), (2, 2).







Rotation

Rotation turns a shape around a fixed point called the centre of rotation.

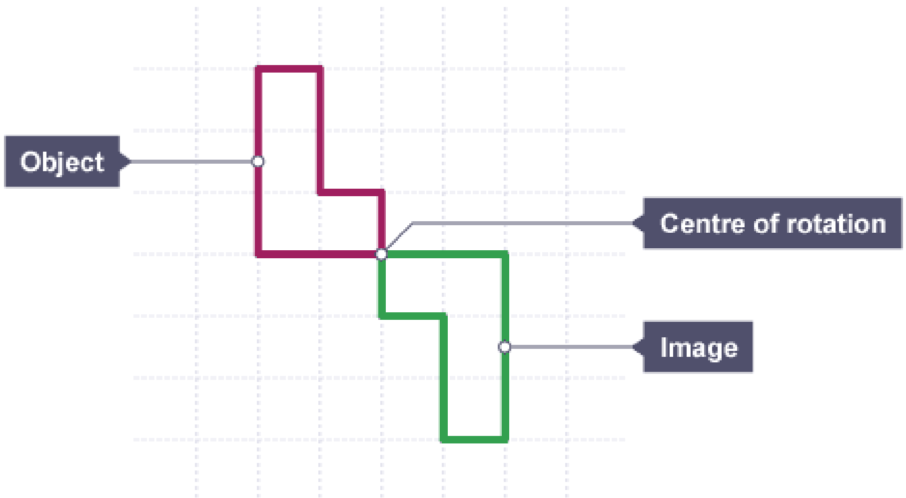
Rotation is an example of a transformation. A transformation is a way of changing the size or position of a shape.

Three pieces of information are needed to rotate a shape:

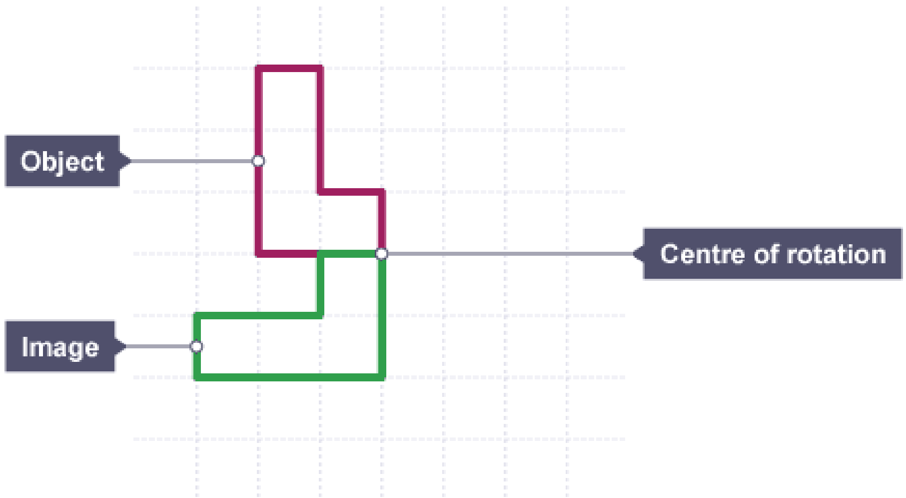
- the centre of rotation

- the angle of rotation

- the direction of rotation

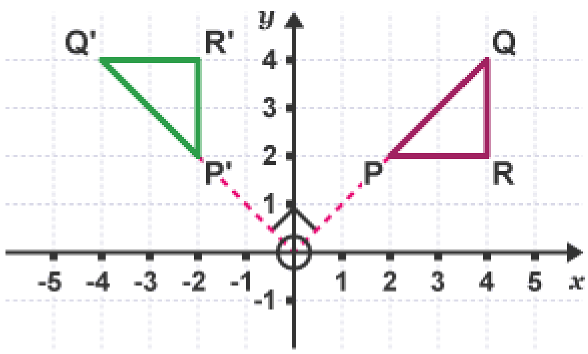


The shape has been rotated 180° about the centre of rotation

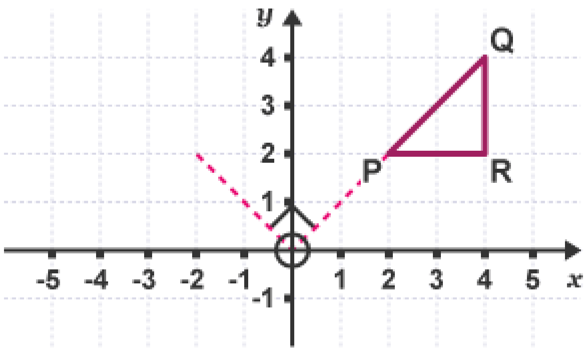


The shape has been rotated 90° anticlockwise about the centre of rotation

For example, the triangle PQR has been rotated 90° anticlockwise about the origin O to create the image P'Q'R'.



Example Rotate the triangle PQR 90° anticlockwise about the origin.

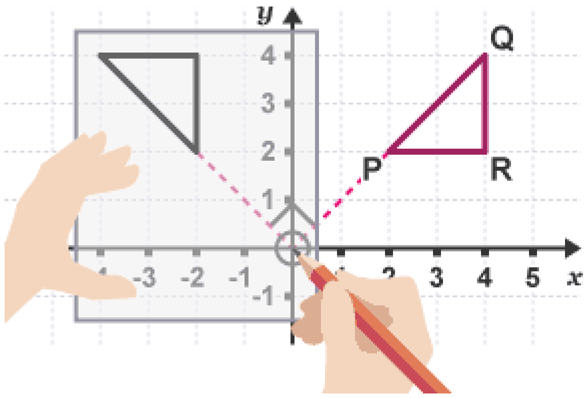


Tracing paper can be used to rotate a shape.

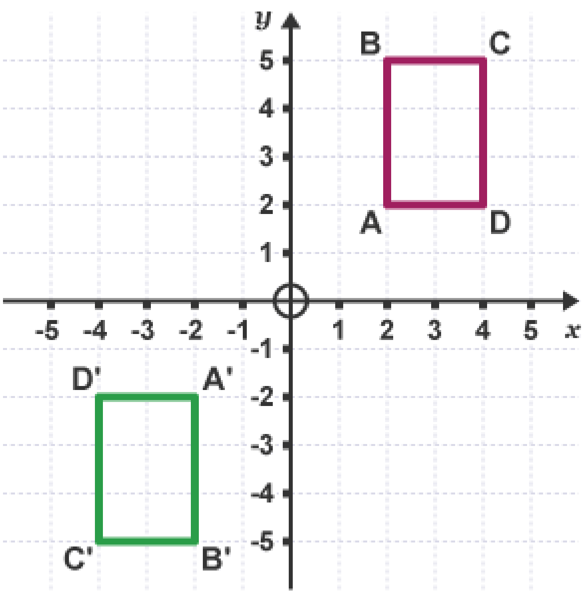
Trace the shape and the centre of rotation.

Hold down the tracing paper with a pencil on the centre of rotation.

Rotate the tracing paper and copy the image.



Question Describe the transformation of the rectangle ABCD.

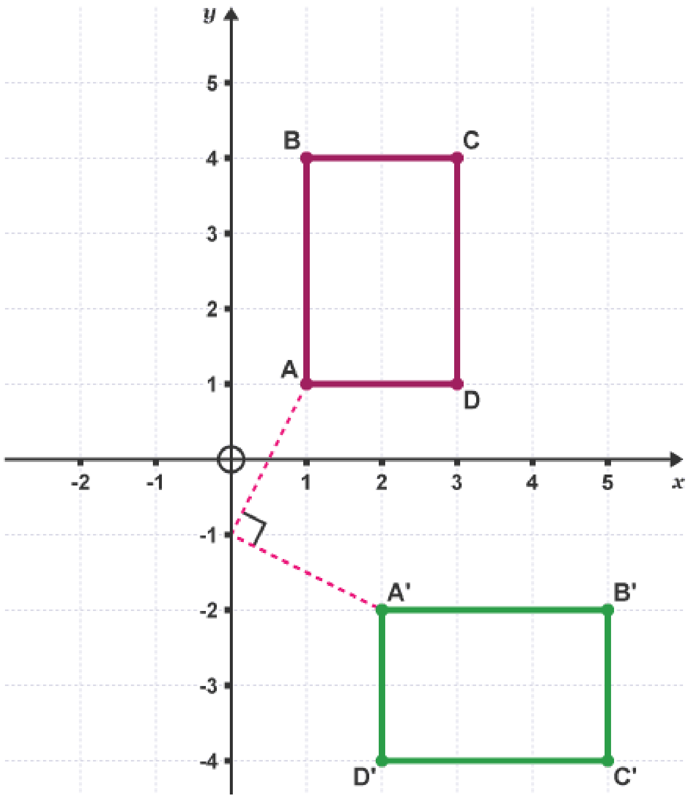


Each vertex of the image A'B'C'D' is the same distance from the origin as the original shape. The origin is the centre of rotation.

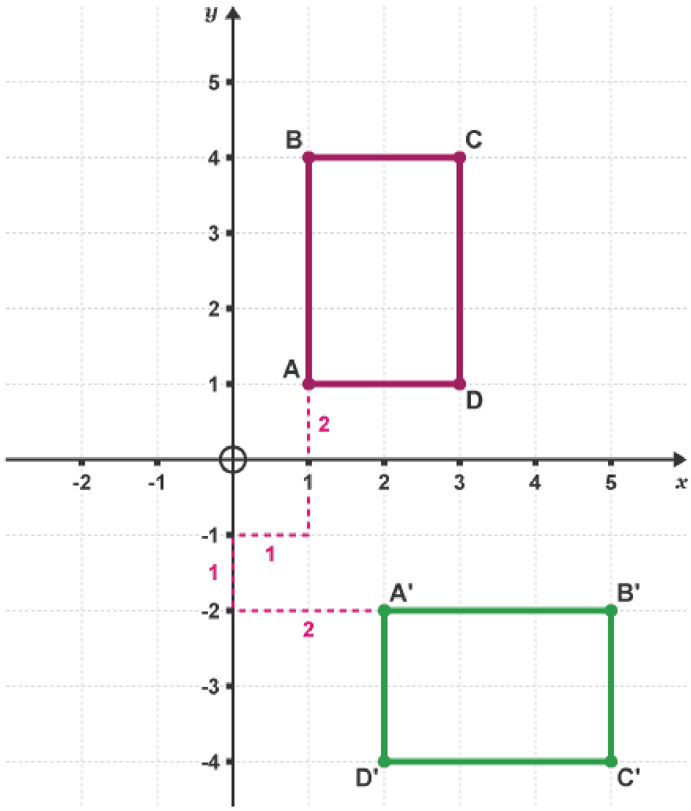
The rectangle ABCD has been rotated 180° about the origin (the direction is not required because it can be either clockwise or anticlockwise).

The centre of rotation may not be at the origin.

Example Rotate the rectangle ABCD 90° clockwise about the point (0,-1).



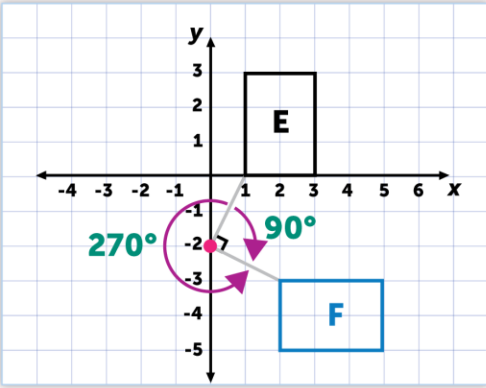
Each corner of the image A'B'C'D' is the same distance from the centre of rotation as the original shape.



Rotation

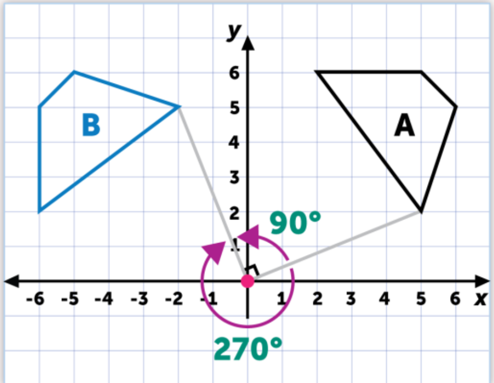
**A rotation turns a shape about a fixed point.
To perform a rotation, three details are needed:**

1 The centre of rotation **2** The angle of rotation **3** The direction of rotation



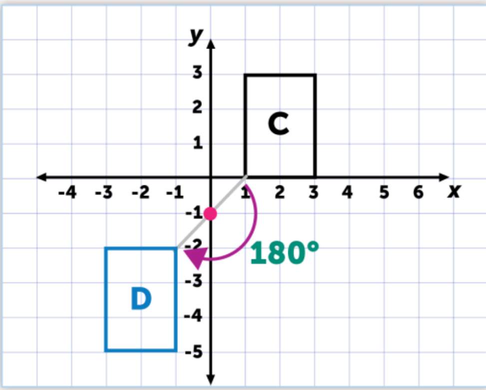
Shape **F** is a rotation of shape **E**
90° clockwise about (0,-2).

Shape **F** is also a rotation of shape **E**
270° anticlockwise about (0, -2).

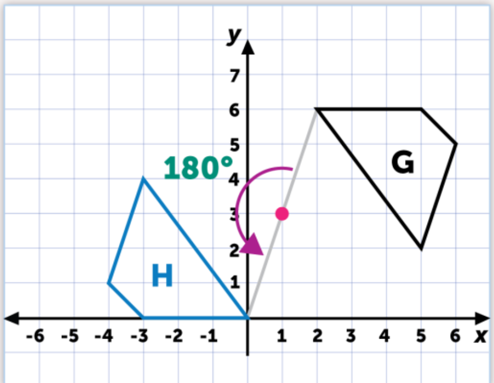


Shape **B** is a rotation of shape **A**
90° anticlockwise about the origin (0,0).

Shape **B** is also a rotation of shape **A**
270° clockwise about the origin (0,0).



Shape **D** is a rotation of shape **C**
180° about (0,-1).



Shape **H** is a rotation of shape **G**
180° about (1,3).

When rotating through 180 degrees, there is no need to specify whether the direction is clockwise or anticlockwise. However, for other angles it is important to specify whether the rotation is clockwise or anticlockwise.

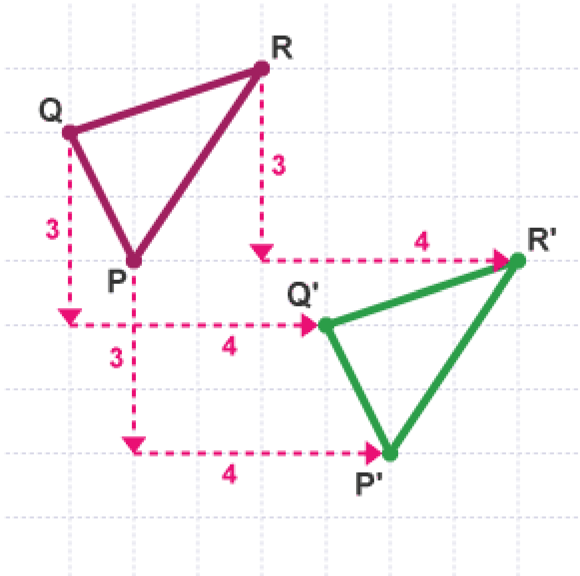
In a rotation, the shapes are congruent. This means that one shape can be turned, flipped or moved so it fits exactly on the other.

Note that 270^0 clockwise could be worked out as 90^0 anticlockwise and 270^0 anticlockwise could be worked out as 90^0 clockwise.

Translation

A translation moves a shape up, down or from side to side but it does not change its appearance in any other way.

Every point in the shape is translated the same distance in the same direction.



Column vectors are used to describe translations.

$\binom{4}{-3}$ means translate the shape 4 squares to the right and 3 squares down.

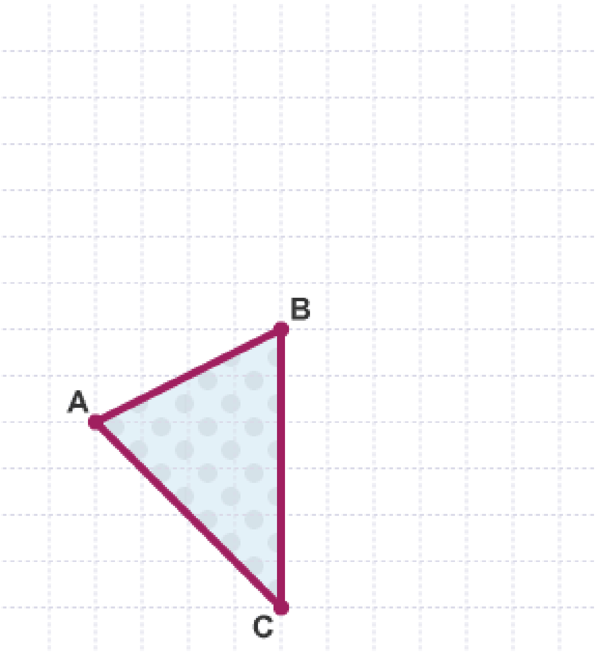
$\binom{-2}{1}$ means translate the shape 2 squares to the left and 1 square up.

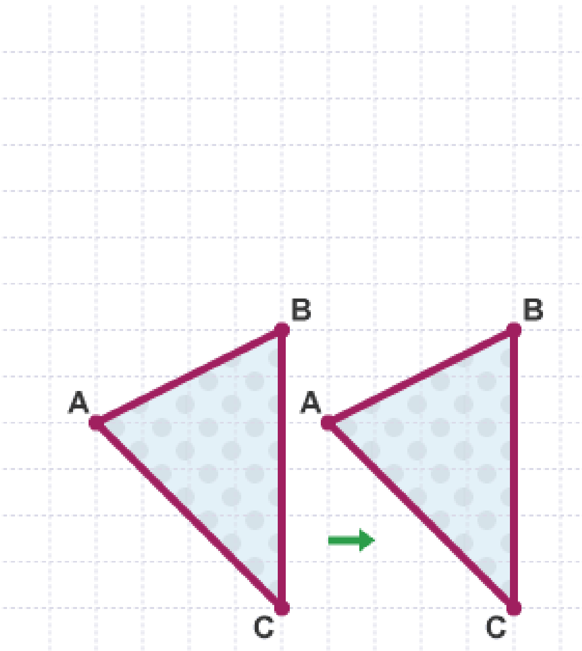
Vectors are given in the form $\binom{x}{y}$ where x is the movement horizontally and y is the movement vertically.

A positive value of x means a movement to the right and a negative value of x means a movement to the left.

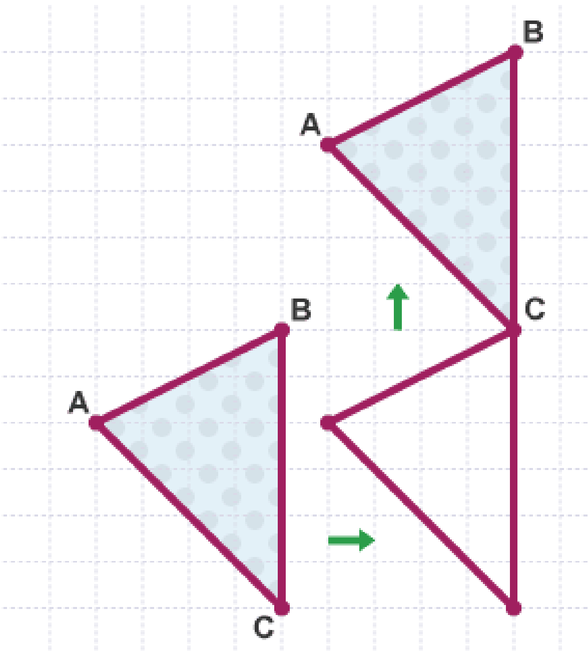
A positive value of y means a movement upwards and a negative value of y means a movement downwards.

Example Translate triangle ABC $\binom{5}{6}$



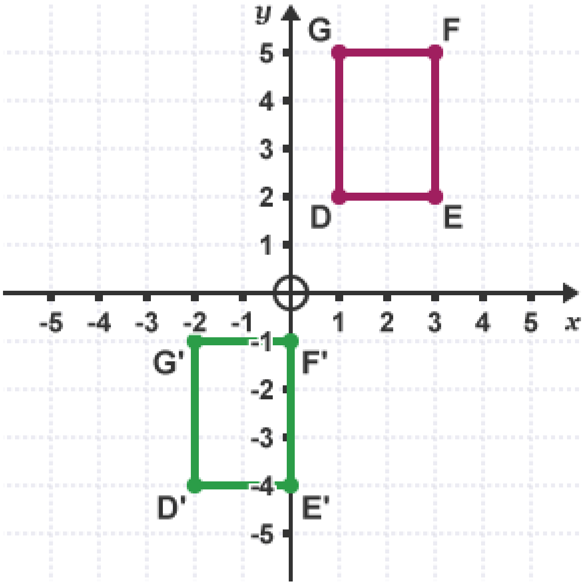


Shape is translated 5 units to the right.



Shape is then translated 6 units up.

Example Describe the transformation of the shape DEFG.



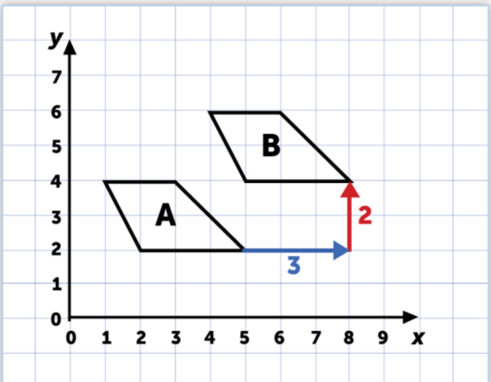
The shape has moved three units to the left and six units down.

The shape has been translated by the vector $\begin{pmatrix}-3\\-6\end{pmatrix}$.

Translation

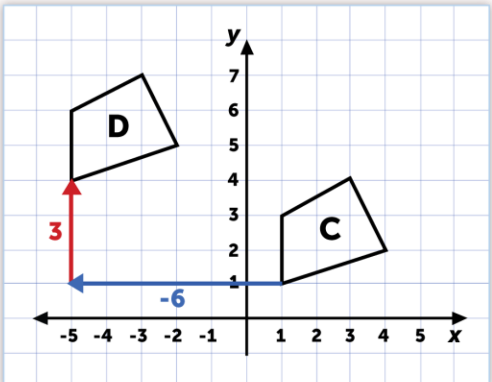
A translation moves every point on a shape the same distance in the same direction.

Shape A has been translated +3 units along the *x*-axis and +2 units up the *y*-axis.



The translation A to B expressed as a vector is $\begin{bmatrix}3\\2\end{bmatrix}$.

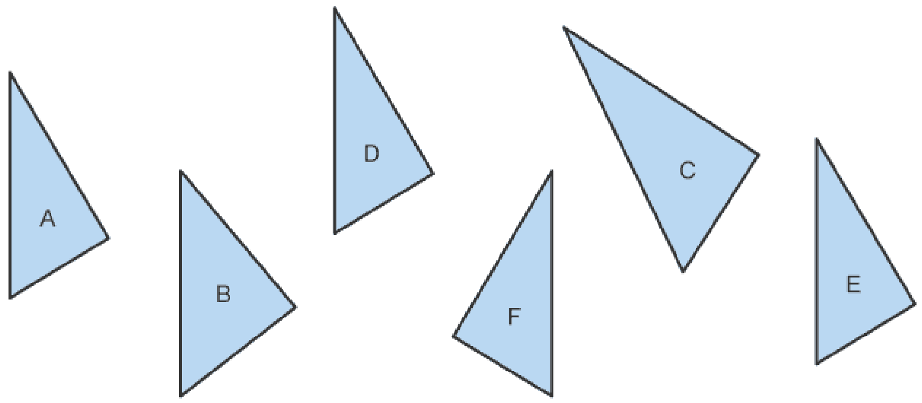
Shape C has been translated -6 units along the *x*-axis and +3 units up the *y*-axis.



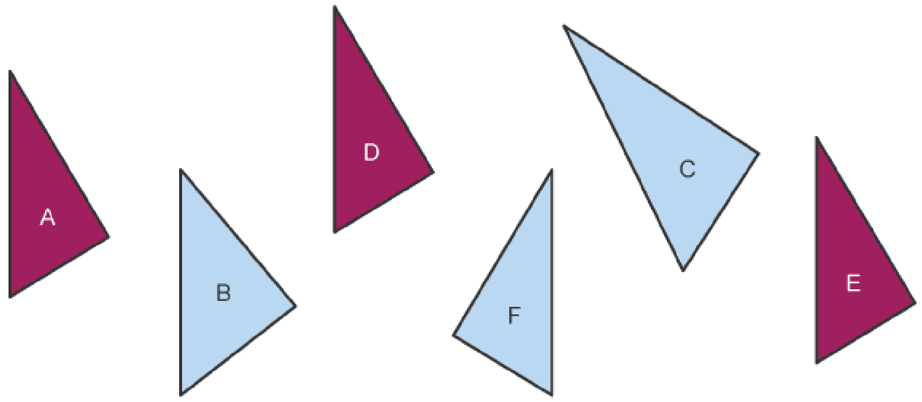
The translation C to D expressed as a vector is $\begin{bmatrix}-6\\3\end{bmatrix}$.

In a translation, the shapes are congruent. This means that one shape can be turned, flipped or moved so it fits exactly on the other.

Example Which shapes are translations of triangle A?



Only D and E are translations of triangle A.



(B is enlarged, F is reflected while C is enlarged and rotated.)

IMPORTANT

When describing:

- **reflections**

write **reflected on the x-axis; reflected on the line y = x, reflected on the line x = 3,** etc.

- **rotations**

write **rotated 90^0 clockwise about the point (0,0); rotated 180^0 about point O;** etc.

- **translations**

write **translated by the vector** $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$; **translated by the vector** $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$, etc.